
5
MULTIPLICATION AND PROPORTIONS

CHAPTER 5 Multiplication and Proportions

In Chapter 2, you focused on simplifying expressions by adding and subtracting like terms. In Section 5.1, you will focus on multiplying expressions. You will also solve equations that contain products. While these new ideas will be introduced using algebra tiles, you will also develop a method to multiply expressions without using tiles.

Then in Section 5.2, you will continue your study of proportional situations started in Section 2.2. By the end of this chapter, you will develop an algebraic method to solve problems involving proportional relationships.

In this chapter, you will learn:

Guiding Questions

Think about these questions throughout this chapter:

What is the area?

How can I write it?

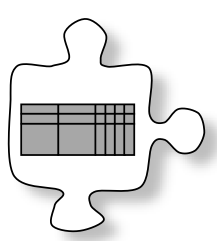
What's the relationship?

How can I solve it?

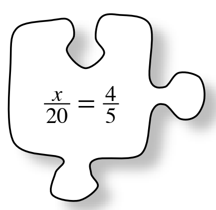
Is there another way?

- How to distribute an expression with and without algebra tiles.
- How to multiply binomials and trinomials using algebra tiles and a generic rectangle.
- How to use the Distributive Property to rewrite expressions and solve equations.
- How to solve multi-variable equations for one of the variables.
- How to write and solve equations with equivalent ratios to solve problems involving proportional relationships.

Chapter Outline



Section 5.1 Using algebra tiles and generic rectangles, you will develop a method to rewrite products, such as $(3x-2)(4+x)$. Then, continuing the solving focus of Chapter 3, you will study how to solve one-variable equations containing products and how to solve multi-variable equations for one of the variables.



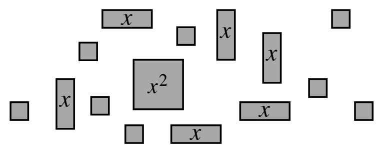
Section 5.2 Here you will continue your study of proportional situations started in Section 2.2. You will develop algebraic techniques to solve proportions and continue to build intuition about what makes a relationship proportional.

5.1.1 What can I do with rectangles?

Exploring an Area Model

In Chapter 2, you used tiles to rewrite algebraic expressions involving addition and subtraction. In this chapter, you will use algebra tiles again, but this time you will rewrite expressions using multiplication.

5-1. Your teacher will put this group of tiles on the overhead:



a. Using your own tiles, arrange the same group of tiles into one large rectangle. On your paper, sketch what your rectangle looks like.

b. What are the dimensions (length and width) of the rectangle you made? Label your sketch with its dimensions.

c. Write a *length* · *width* = *area* statement showing the equivalence of the area as the **product** of its length and width and as the **sum** of its parts.

5-2. Your teacher will assign several of the expressions below. For each expression, build a rectangle using all of the tiles, if possible. Sketch each rectangle, find its dimensions, and write an expression showing the equivalence of the area as a **sum** (like x^2+5x+6) and as a **product** (like $(x+3)(x+2)$). If it is not possible to build a rectangle, explain why not.

a. x^2+3x+2

b. $6x+15$

c. $2x^2+7x+6$

d. $xy+x+y+1$

e. $2x^2+10x+12$

f. $2y^2+6y$

g. $y^2+xy+2x+2y$

h. $3x^2+4x+1$

i. $x^2+2xy+y^2+3x+3y+2$

j. $2xy+4y+x+2$

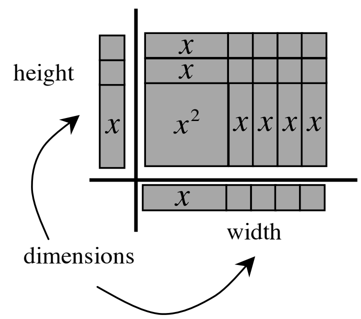
5-3. Make a rectangle from any number of tiles. Your rectangle must contain at least one of each of the following tiles: x^2, y^2, xy, x, y, and 1. Sketch your rectangle in your Learning Log and write its area as a **product** and as a **sum**. Explain how you know that the product and sum are equivalent. Title this entry "Area as a Product and as a Sum" and label it with today's date.

METHODS AND MEANINGS

MATH NOTES

Multiplying Algebraic Expressions with Tiles

The area of a rectangle can be written two different ways. It can be written as a **product** of its base and height or as a **sum** of its parts. For example, the area of the shaded rectangle at right can be written two ways:

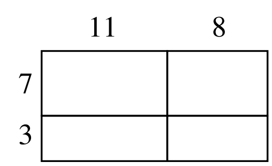


area as a product **area as a sum**

$$\underbrace{(x+4)}_{\textit{base}}\underbrace{(x+2)}_{\textit{height}} = \underbrace{x^2+6x+8}_{\textit{area}}$$

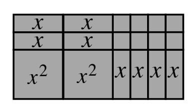
Review & Preview

5-4. For the entire rectangle at right, find the area of each part and then find the area of the whole.



5-5. A tile pattern has 5 tiles in Figure 0 and adds 7 tiles in each new figure. Write the equation of the line that represents the growth of this pattern.

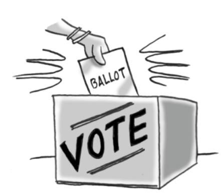
5-6. Write the area of the rectangle at right as a **product** and as a **sum**. Refer to the Math Notes box for this lesson if you need help.



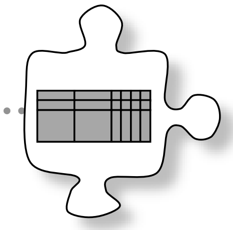
5-7. Draw Figures 1, 2, and 3 for a tile pattern that could be described by $y=-3x+10$.

5-8. Fisher thinks that any two lines must have a point of intersection. Is he correct? If so, explain how you know. If not, produce a **counterexample**. That is, find two lines that do not have a point of intersection and explain how you know.

5-9. In the last election, candidate A received twice as many votes as candidate B. Candidate C received 15,000 fewer votes than candidate B. If a total of 109,000 votes were cast, how many votes did candidate A receive?



5.1.2 How can I rewrite a product?

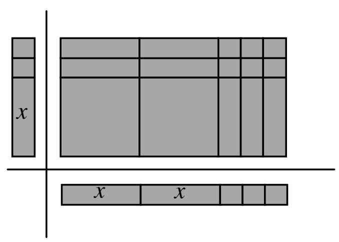


Multiplying Binomials and the Distributive Property

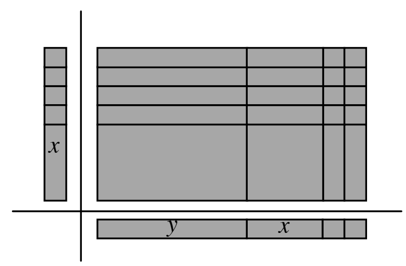
In Lesson 5.1.1, you made rectangles with algebra tiles and found the dimensions of the rectangles. You wrote the area both as a sum and as a product. Today you will **reverse** the process, starting with the product of the dimensions of a rectangle and finding its area as a sum.

5-10. For each of the following rectangles, find the dimensions (length and width) and write the area as the **product** of the dimensions and as the **sum** of the tiles. Remember to combine like terms whenever possible.

a.



b.



5-11. Your teacher will assign your team four of the expressions below. Use your cornerpiece to build rectangles with the given dimensions. Sketch each rectangle on your paper, label its dimensions, and write an equivalence statement for its area as a **product** and as a **sum**. Be prepared to share your solutions with the class.

a. $(2x)(4x)$

b. $(x+3)(2x+1)$

c. $2x(x+5)$

d. $(2x+1)(2x+1)$

e. $x(2x+y)$

f. $(2x+5)(x+y+2)$

g. $2(3x+5)$

h. $y(2x+y+3)$

5-12. With the class, examine the solutions you found for parts (c), (e), (g), and (h) of problem 5-11. As you discuss your observations, you may want to focus on these questions:

Do you see a pattern?

What happens to the term outside the parentheses?

What happens to the terms inside the parentheses?

Does this pattern make sense?

5-13. Using the patterns your team identified, multiply the following expressions *without* using your tiles. Be ready to share your process with the class.

a. $2x(6x+5)$

b. $6(4x+1)$

c. $3y(4x+3)$

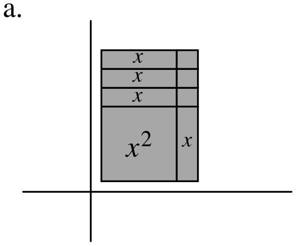
d. $7y(10x+11y)$

5-14. The pattern you used to multiply a one-term expression (like x) by a multiple-term expression (like $x+2$) is called the **Distributive Property**. In your Learning Log, describe this pattern. Make up your own example and show the pattern in as many ways as you can. Title this entry "The Distributive Property" and label it with today's date.

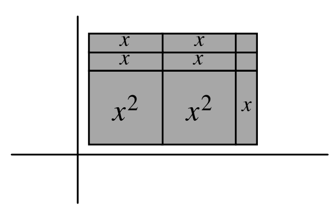
Review & Preview

5-15. Examine the rectangles formed with tiles below. For each figure, write its area as a **product** of the base and height and as a **sum** of its parts.

a.

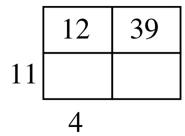


b.

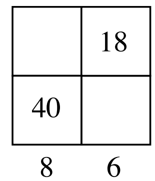


5-16. Find the total area of each rectangle below. Each number inside the rectangle represents the area of that smaller rectangle, while each number along the side represents the length of that portion of the side.

a.



b.



5-17. When solving $\frac{x}{6}=\frac{5}{2}$ for x, Nathan noticed that x is divided by 6.

a. What can he do to both sides of the equation to get x alone?

b. Solve for x. Then check your solution in the original equation.

c. Use the same process to solve this equation for x: $\frac{x}{10}=\frac{2}{5}$.

5-18. Jamila wants to play a game called "Guess My Line." She gives you the following hints: "Two points on my line are (1, 1) and (2, 4)."

a. What is the growth rate of her line? A graph of the line may help.

b. What is the y-intercept of her line?

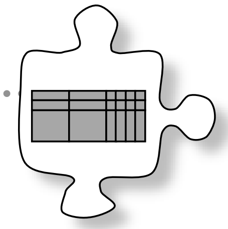
c. What is the equation of her line?

5-19. A calculator manufacturer offers two different models for students. The company has sold 10,000 scientific calculators so far and continues to sell 1500 per month. It has sold 18,000 graphical models and continues to sell 1300 of this model each month. When will the sales of scientific calculators equal the sales of graphical calculators?

5-20. On graph paper, make an $x \rightarrow y$ table and graph $y=2x^2-x-3$. Find its x- and y-intercepts.

5.1.3 How can I generalize the process?

Using Generic Rectangles to Multiply



You have been using algebra tiles and the concept of area to multiply algebraic expressions. Today you will be introduced to a tool that will help you find the product of the dimensions of a rectangle. This will allow you to multiply expressions without tiles.

5-21. Use the Distributive Property to find each product below.

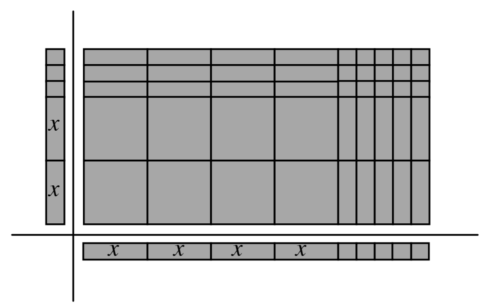
a. $6(-3x+2)$

b. $x(4x-2)$

c. $5t(10-3t)$

d. $-4(8-6k+y)$

5-22. Write the area as a **product** and as a **sum** for the composite rectangle shown at right.



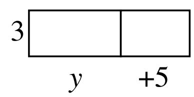
5-23. Now examine the following diagram. How is it similar to the set of tiles in problem 5-22? How is it different? Talk with your teammates and write down all of your observations.

3	$12x$	15
$2x$	$8x^2$	$10x$
	$4x$	5

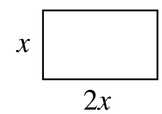
5-24. Diagrams like the one in problem 5-23 are referred to as **generic rectangles**. Generic rectangles allow you to use an area model to multiply expressions without using the algebra tiles. Using this model, you can multiply with values that are difficult to represent with tiles.

Draw each of the following generic rectangles on your paper. Then find the area of each part and write the area of the whole rectangle as a **product** and as a **sum**.

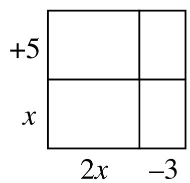
a.



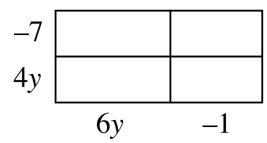
b.



c.



d.



e. How did you find the area of the individual parts of each generic rectangle?

5-25. Multiply and simplify the following expressions using either a generic rectangle or the Distributive Property. For part (a), verify that your solution is correct by building a rectangle with algebra tiles.

a. $(x+5)(3x+2)$

b. $(2y-5)(5y+7)$

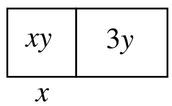
c. $3x(6y-11)$

d. $(5w-2p)(3w+p-4)$

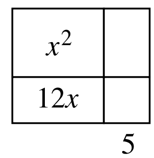
5-26. THE GENERIC RECTANGLE CHALLENGE

Copy each of the generic rectangles below and fill in the missing dimensions and areas. Then write the entire area as a product and as a sum. Be prepared to share your reasoning with the class.

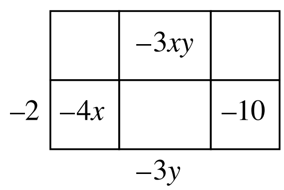
a.



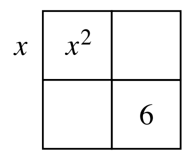
b.



c.



d.



METHODS AND MEANINGS

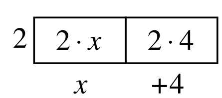
MATH NOTES

The Distributive Property

The **Distributive Property** states that for any three terms a, b, and c:

$$a(b+c) = ab + ac$$

That is, when a multiplies a group of terms, such as $(b + c)$, then it multiplies *each* term of the group. For example, when multiplying $2(x+4)$, the 2 multiplies both the x and the 4. This can be shown with **algebra tiles** or in a **generic rectangle** (see below).



$$2(x+4) = 2 \cdot x + 2 \cdot 4 = 2x + 8$$

The 2 multiplies each term.

5-27. Use a generic rectangle to multiply the following expressions. Write each solution both as a sum and as a product.

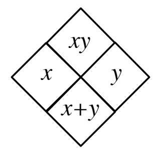
a. $(2x+5)(x+6)$ b. $(m-3)(3m+5)$

c. $(12x+1)(x-5)$ d. $(3-5y)(2+y)$

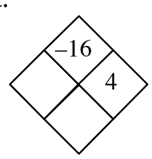
5-28. Solve each equation below for x. Then check your solutions.

a. $\frac{x}{8}=\frac{3}{4}$ b. $\frac{2}{5}=\frac{x}{40}$ c. $\frac{1}{8}=\frac{x}{12}$ d. $\frac{x}{10}=\frac{12}{15}$

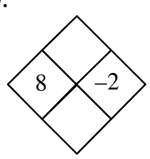
5-29. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



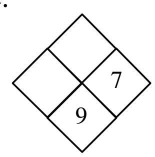
a.



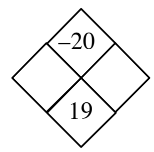
b.



c.



d.



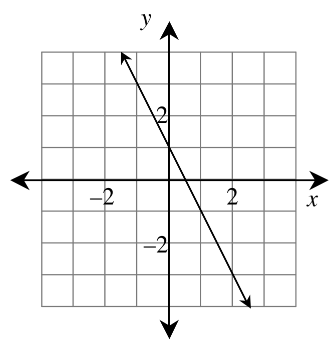
5-30. Review what you know about graphs by answering the following questions.

a. Find the equation of the line graphed at right.

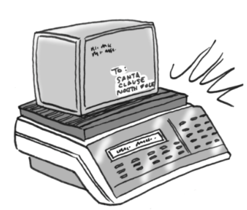
b. What are its x- and y-intercepts?

c. On your own graph paper, graph the line.

d. On the same set of axes, graph a line *parallel* to the line graphed at right, but through the *origin* (0, 0). Find the equation of this new line.



5-31. Mailboxes Plus sends packages overnight for \$5 plus \$0.25 per ounce. United Packages charges \$2 plus \$0.35 per ounce. Mr. Molinari noticed that his package would cost the same to mail using either service. How much does his package weigh?



5-32. Decide if the statement below is true or false. **Justify** your response.

"The expression $(x+3)(x-1)$ is equivalent to $(x-1)(3+x)$."

5.1.4 What if an equation has a product?

Solving Equations With Multiplication

Now that you know how to multiply algebraic expressions, you can solve equations that involve multiplication.

5-33. Review what you learned in Lesson 5.1.3 by multiplying each expression below. First decide if you will multiply each expression using the Distributive Property or using a generic rectangle. Remember to simplify your result.

a. $(6x-11)(2x+5)$

b. $-2x(15x-3)$

c. $(6-y)(y+2)$

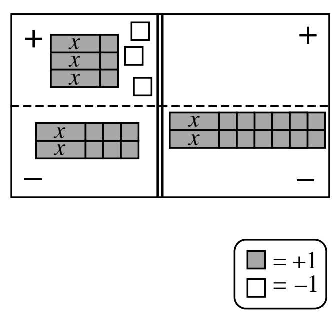
d. $16(3-m^2)$

5-34. How can you represent $3(2x+1)$ with algebra tiles? Work with your team to build this expression.

a. Build an equation mat to represent the equation $3(2x+1)=8x-5$. Solve this equation and record your steps algebraically.

b. Check that your solution is correct by substituting your answer into the original equation.

5-35. **Multiple Choice:** Which equations below are represented by the diagram at right? Be prepared to defend your answer.



a. $3(x+1)-3-2(x+3)=-2(x+6)$

b. $3x-1-3-2x+6=-2x+12$

c. $3x+3-3-2x-6=-2x-12$

5-36. Copy one of the correct equations from problem 5-35 and solve for x. Be sure to record all of your steps. Check your solution by substituting your answer into your equation.

5-37. Your teacher will assign you several of the equations below. Work with a partner to solve the equations using algebra tiles and an equation mat. Check your solution by substituting your answer into the original equation.

a. $3(x-4)=15$

b. $1-2(3x-5)=11$

c. $5(y-4)=10$

d. $-2(x-2)=11$

e. $6(x+4)=3(5x+2)$

f. $5-x(x+3)=-(x+5)(x+1)$

5-38. Now work with your team to solve each of these equations without using tiles. You may want to draw generic rectangles to help you rewrite the products.

a. $2(y-2)=-6$

b. $43=4(x+6)-1$

c. $(x+3)(x+4)=(x+1)(x+2)$

d. $2(x+1)+3=3(x-1)$

METHODS AND MEANINGS

MATH NOTES

Checking a Solution

To check a solution to an equation, substitute the solution into the equation and verify that it makes the two sides of the equation equal.

For example, to verify that $x=10$ is a solution to the equation $3(x-5)=15$, substitute 10 into the equation for x and then verify that the two sides of the equation are equal.

As shown at right, $x=10$ is a solution to the equation $3(x-5)=15$.

$$3(10-5)\stackrel{?}{=}15$$
$$3(5)\stackrel{?}{=}15$$
$$15=15$$

Correct!

What happens if your answer is incorrect? To investigate this, test any solution that is not correct. For example, try substituting $x=2$ into the same equation. The result shows that $x=2$ is not a solution to this equation.

$$3(2-5)\stackrel{?}{=}15$$
$$3(-3)\stackrel{?}{=}15$$
$$-9\neq 15$$

Not true, so x = 2 is not a solution.

Review & Preview

5-39. Which equation below has *no* solution? Explain how you know.

a. $4(x+1) = 2x + 4$

b. $9 - 5x + 2 = 4 - 5x$

5-40. Rena says that if $x = -5$, the equation below is true. Her friend, Dean, says the answer is $x = 3$. Who is correct? **Justify** your conclusion.

$$9(x+4) = 1 + 2x$$

5-41. Find the rule for the pattern represented at right.

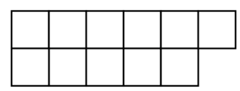
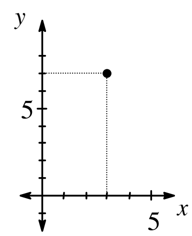
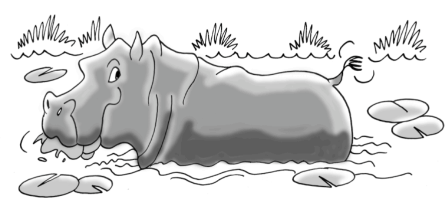


Figure 1



5-42. Harry the Hungry Hippo is munching on the lily pads in his pond. When he got to the pond, there were 20 lily pads, but he is eating 4 lily pads an hour. Heinrick the Hungrier Hippo found a better pond with 29 lily pads! He eats 7 lily pads every hour.



a. If Harry and Heinrick start eating at the same time, when will their ponds have the same number of lily pads remaining?

b. How many lily pads will be left in each hole at that time?

5-43. Graph each equation below on the same set of axes and label the point of intersection with its coordinates.

$$y = 2x + 3 \qquad\qquad y = x + 1$$

5-44. Shooter Magee is the Wolverines' best free-throw shooter. He normally makes three out of every four shots. In an upcoming charity event, Shooter will shoot 500 free throws. If he makes over 400 baskets, the school wins $1000. Should the Wolverines expect to win the cash for the school? Show and organize your work.

5.1.5 How can I change it to $y = mx + b$ form?

Working With Multi-Variable Equations

So far in this course, you have used your equation mat to find solutions for all types of linear equations with one variable. Today you will learn how to **apply** these skills to solving linear equations with two variables. As you work today, keep the following questions in mind:

What is a solution to an equation? What does it look like?

What is the growth factor?

What is the y-intercept?

5-45. You now have a lot of experience working with equations that compare two quantities. For example, while working with the height of a tree, you found the relationship $y = 4x + 5$, which compared x (the number of years after it was planted) with y (its height in feet). For this tree:

a. What was its starting height? How can you tell from the equation?

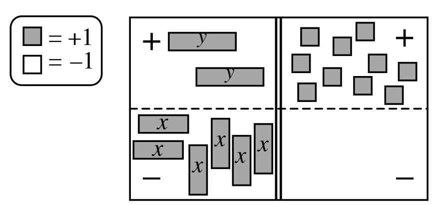
b. What was its growth rate? That is, how many feet did the tree grow per year? **Justify** your answer.

5-46. CHANGING FORMS

You could find the growth rate and starting value for $y = 4x + 5$ quickly because the equation is in $y = mx + b$ form. But what if the equation is in a different form? Explore this situation below.

a. The line $-6x + 2y = 10$ is written in **standard form**. Can you tell what the growth rate of the line is? Its y-intercept? Predict these values.

b. The equation $-6x + 2y = 10$ is shown on the equation mat at right. Set up this equation on your equation mat using tiles. Using only "legal" moves, rearrange the tiles to get y by itself on the left side of the mat. Record each of your moves algebraically.



c. Now use your result from part (b) to find the growth factor and y-intercept of the line $-6x + 2y = 10$. Did your result match your prediction in part (a)?

5-47. Your teacher will assign you one of the linear equations listed below. For your equation:

- Use algebra tiles to set up the equation on your equation mat.
- Using only "legal" moves, rearrange your tiles to create an equation that starts with "$y = \ldots$" Be sure to record all of your moves algebraically and be prepared to share your steps with the class.
- What is the growth factor of your line? What is the y-intercept? How can you tell?

a. $2x + y = 3x - 7$

b. $x + 2y = 3x + 4$

c. $3y + 2 = 2y - 5x$

d. $2(y - 3) = 2x - 6$

e. $5 - 3(x + 1) = 2y - 3x + 2$

f. $x - (y + 2) = 2(2x + 1)$

5-48. Solve each of the following equations for the indicated variable. Use your equation mat if it is helpful. Write down each of your steps algebraically.

a. Solve for y: $2(y - 3) = 4$

b. Solve for x: $2x + 5y = 10$

c. Solve for y: $6x + 3y = 4y + 11$

d. Solve for x: $3(2x + 4) = 2 + 6x + 10$

e. Solve for x: $y = -3x + 6$

f. Solve for p: $m = 8 - 2(p - m)$

g. Solve for y: $x^2 + 4y = (x + 6)(x - 2)$

h. Solve for q: $4(q - 8) = 7q + 5$

MATH NOTES

METHODS AND MEANINGS

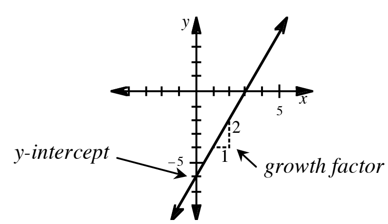
Linear Equations

A **linear equation** is an equation that forms a line when it is graphed. This type of equation may be written in several different forms. Although these forms look different, they are equivalent; that is, they all graph the same line.

STANDARD FORM: An equation in $ax + by = c$ form, such as $-6x + 3y = 18$.

$\boldsymbol{y = mx + b}$ **FORM:** An equation in $y = mx + b$ form, such as $y = 2x - 6$.

You can quickly find the **growth factor** and ***y*-intercept** of a line in $y = mx + b$ form. For the equation $y = 2x - 6$, the growth factor is 2, while the y-intercept is (0, –6).



$$y = 2x - 6$$

growth factor *y-intercept*

Review & Preview

5-49. Use what you know about $y = mx + b$ to graph each of the following equations quickly on the same set of axes.

a. $y = 3x + 5$ b. $y = -2x + 10$ c. $y = 1.5x$

5-50. Multiply each of the following expressions. Show all of your work.

a. $(x + 3)(4x + 5)$ b. $(-2x - 4)(3x + 4)$

c. $(3y - 8)(-x + y)$ d. $(y - 4)(3x + 5y - 2)$

5-51. Solve each of the following equations for the indicated variable. Show all of your steps.

a. $y = 2x - 5$ for x

b. $p = -3w + 9$ for w

c. $2m - 6 = 4n + 4$ for m

d. $3x - y = -2y$ for y

5-52. Find the rule for the following tile pattern.

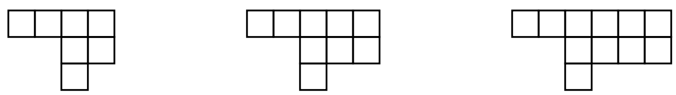


Figure 2 Figure 3 Figure 4

5-53. Consider these two equations:

$$y = 3x - 2$$
$$y = 4 + 3x$$

a. Graph both equations on the same set of axes.

b. Solve this system using the Equal Values Method.

c. Explain how the answer to part (b) agrees with the graph you made in part (a).

5-54. Joe drove 100 miles from San Francisco to Gilroy and used 4 gallons of gas. How much gas should he expect to use for a 3000-mile trip to New York City? Be sure to **justify** your reasoning.

5.1.6 What kinds of equations can I solve now?

Solving Equations Without Manipulatives

So far, you have developed your equation-solving skills in three major sections of this course (Sections 2.1, 3.2, and 5.1). Today you will practice solving equations while moving away from using algebra tiles. At the end of the lesson, you will summarize everything you know about solving equations.

5-55. Your teacher will explain the way you are working today on the problems below. As you work, be sure to record all of your steps carefully. Check your solutions, if possible.

a. Solve for x: $5(4x+3)=75$

b. Solve for y: $x-2y=4$

c. Solve for x: $-6=-6(3x-8)$

d. Solve for y: $3x+6y=24$

e. Solve for x: $2-3(2x-1)=17$

f. Solve for y: $5+2(x+y)=11$

g. Solve for x: $y=-3x+4$

h. Solve for x: $x(2x-1)=2x^2+5x-12$

i. Solve for w: $2(v-3)=1-(w+4)$

j. Solve for x: $4x(x+1)=(2x-3)(2x+5)$

5-56. SUMMARY OF SOLVING EQUATIONS

Write a letter to Clarissa, a new student in class, explaining everything you have learned about how to solve equations. Clarissa does not have algebra tiles, so you will need to show her how to solve *without* the tiles. Make up examples that show all of the different equation-solving skills you have. Be sure to explain your ideas to her thoroughly so she will know what to do on her own.

Review & Preview

5-57. Solve each of the following equations. Be sure to show your work carefully and check your answers.

a. $2(3x-4)=22$

b. $6(2x-5)=-(x+4)$

c. $2-(y+2)=3y$

d. $3+4(x+1)=159$

5-58. Find the dimensions of the generic rectangle at right. Then write an equivalency statement (length · width = area) of the area as a product and as a sum.

x^2	$-5x$
$3x$	-15

5-59. Graph $y=-2x^2+4$. Be sure to decide on a reasonable scale and label your axes clearly.

5-60. One number is five more than a second number. The product of the numbers is 3300. Find the two numbers.

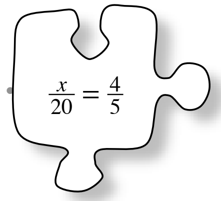
5-61. Ms. B and Ms. D are writing problems for an algebra book. Ms. B started with 10 problems already written, and she can write 6 problems an hour. Ms. D had no problems written, but she writes 10 problems an hour.

a. When will Ms. B have the same number of problems written as Ms. D?

b. How many problems will they each have written at that time?

5-62. How many yearbooks should your school order? Your student government surveyed three homeroom classes, and 55 of 90 students said that they would definitely buy a yearbook. If your school has 2000 students, approximately how many books should be ordered? Show and organize your work.

5.2.1 How can I write proportions?



Setting Up and Solving Proportions

In Chapter 2, you studied proportional situations and used several strategies to solve problems involving such situations. Since then, you have learned to set up and solve equations to solve many types of problems. Today you will investigate methods for using equations to solve proportion problems.

5-63. Use what you know about solving equations to solve for x. Remember to check your solution to each equation. Be prepared to share your method with the class.

a. $\frac{x}{2} = 9$ b. $\frac{x}{18} = \frac{2}{3}$ c. $\frac{3}{2} = \frac{x+3}{5}$ d. $\frac{7}{3} = \frac{4}{x}$

5-64. POLITICAL POLL

Mr. Mears is running for mayor of Atlanta. His campaign managers are eager to determine how many citizens of Atlanta will vote for him in the upcoming election. They decided to pay a respected, impartial statistical company to survey potential voters (a process called "polling") in order to find out how many people will probably vote for Mr. Mears.

One afternoon, pollsters called 100 random potential voters in Atlanta to ask them how they would vote in the election. During that survey, 68 people indicated that they would vote for Mr. Mears.

a. If the pollsters had instead called 50 randomly selected potential voters, predict how many people would have said that they would vote for Mr. Mears.

b. Is this relationship proportional? Why or why not?

c. Carina decided to organize the information in a table like the one shown at right. She wants to figure out how many people will probably vote for Mr. Mears if 350,125 people vote in the election. Help her determine how many votes Mr. Mears will probably receive. Then complete her table on your paper. Be prepared to share your method with the class.

Number of Potential Voters	**Number of Votes Expected for Mr. Mears**
50	
100	68
350,125	

5-65. Carina noticed a pattern in her table. If she makes a **ratio** (a fraction) of the two numbers of potential voters and another ratio of the two numbers of votes for Mr. Mears, the two ratios are equal! See her notes below:

$\frac{50 \text{ number of voters}}{100 \text{ number of voters}} = \frac{1}{2}$

Number of Potential Voters	Number of Votes Expected for Mr. Mears
50	34
100	68

$\frac{34 \text{ votes for Mears}}{68 \text{ votes for Mears}} = \frac{1}{2}$

a. Carina wonders what would happen if she created ratios with numbers in the same row. Write two ratios using the values in the rows circled at right. Are your ratios equal?

Number of Potential Voters	Number of Votes Expected for Mr. Mears
50	34
100	68

b. What about diagonally? Will the ratios be equal? Set up some ratios and determine if they are equal.

c. Why are some ratios equal and others not?

d. Carina's neighborhood has 527 potential voters. If her neighborhood reflects the entire city, how many neighbors will probably vote for Mr. Mears? Since she does not know the answer to this question, she placed an x in the table at right.

Number of Potential Voters	Number of Votes Expected for Mr. Mears
50	34
100	68
527	x

Write an equation using two equal ratios and solve for x. Then answer her question.

5-66. Make a table and set up an equation for each proportional situation below.

a. In two minutes, Stacie can write her name 17 times. How long will it take her to write her name 85 times?

b. Eight of 29 students in your class want to attend the Winter Ball. If your class represents the entire school, how many of the 1490 students will probably attend the dance?

METHODS AND MEANINGS

MATH NOTES

Ratios and Proportions

A **ratio** is a way to compare two related numbers, such as 68 expected votes for Mr. Mears out of 100 people surveyed. It can be written with a colon, such as 68:100, or it can be written as a fraction, such as:

$$\frac{68 \text{ votes for Mr. Mears}}{100 \text{ people surveyed}}$$

A ratio can compare any two quantities, such as comparing the number of boys and girls in your class (such as 17 boys:18 girls), or comparing the heights of two people (such as $\frac{62 \text{ inches}}{65 \text{ inches}}$).

An equation that sets two ratios equal is called a **proportion**. For example, the proportion below is an equation made up of two equal ratios:

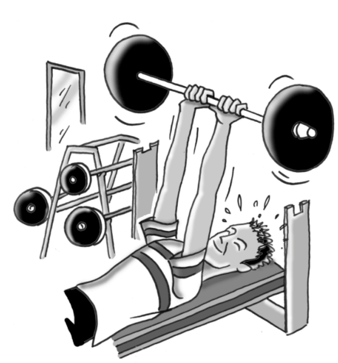
$$\frac{68 \text{ votes for Mr. Mears}}{100 \text{ people surveyed}} = \frac{34 \text{ votes for Mr. Mears}}{50 \text{ people surveyed}}$$

Review & Preview

5-67. Chi loves to read. He can speed-read 40 pages in 3 minutes. How long should it take him to read *The Scarlet Letter*, a 265-page novel?

5-68. GETTING IN SHAPE

Frank weighs 160 pounds and is on a diet to gain two pounds a week so that he can make the football team. John weighs 208 pounds and is on a diet to lose three pounds a week so that he can be on the wrestling team in a lower weight class.



a. If Frank and John can meet these goals with their diets, when will they weigh the same, and how much will they weigh at that time?

b. Clearly explain your method.

5-69. Below are two pairs of equal ratios. For each pair, find two more ratios that are equal.

a. $\frac{1}{5}=\frac{10}{50}=\frac{?}{?}=\frac{?}{?}$

b. $\frac{13}{20}=\frac{65}{100}=\frac{?}{?}=\frac{?}{?}$

5-70. Find each of the following products by drawing and labeling a generic rectangle or by using the Distributive Property.

a. $(x+5)(x+4)$

b. $2y(y+3)$

5-71. Simplify the expressions below. You may want to draw or visualize algebra tiles to help you rewrite these problems.

a. $(2x^2+3x+5)+(x^2+2x+8)$

b. $(3x^2+8x+1)+(2x^2+8x+4)$

c. $(3x^2+5x+7)-(4x^2+x+1)$

d. $(x^2+9x+8)-(x^2+4x+8)$

e. $(7x^2+x+10)-(3x^2+12x+12)$

5.2.2 What strategy can I use to solve?

Practice With Proportions

$\frac{x}{20}=\frac{4}{5}$

In the last lesson, you used equations to solve problems involving proportional situations. Today you will practice writing and solving these special equations, called **proportions**, while you help a student set up a recycling program for her school. As you work, focus on these questions:

What information can you use to answer this question?

How can you use that information to write an equation?

5-72. Solve for x. Remember to check your solution to each equation.

a. $7.5=\frac{x}{4}$

b. $\frac{x}{20}=\frac{4}{5}$

c. $\frac{x-4}{12}=\frac{7}{3}$

d. $\frac{100}{30}=\frac{4}{3x}$

5-73. RECYCLING CLUB

Elsie is starting a recycling club at her school and hopes to use the money earned from recycling cans to buy recycling bins for the school.

Elsie first needs to figure out how much the cans that can be collected at her school will weigh, so she starts by weighing the cans in her recycling bin at home. She finds that 50 cans weigh 0.77 kg. The next day, Elsie counts cans at school and finds that her fellow students throw away 1240 cans each day.

a. Put all of Elsie's information into a table like the one shown at right. Let x represent anything Elsie does not know yet.

# of Cans	Weight (kg)

b. Write and solve a **proportion** (an equation setting two ratios equal) from your table. How much do all of the school's cans weigh?

c. Elsie's school just got a new soda machine in the cafeteria. Now the students at her school consume 2070 cans a day. Add this information to your table, and then use a proportion to find out how much all of those cans weigh.

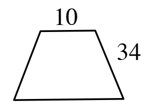
5-74. In order to buy recycling bins for her school, Elsie plans to collect empty aluminum cans in big plastic bags and drive them to a local recycling center. Elsie wants to figure out how many days she will need to use plastic bags before she can buy the recycling bins.

a. The recycling center pays 25 cents per kg for aluminum cans. If her school recycles 2070 cans each day, how much money will Elsie earn each day by recycling?

b. Elsie has a friend at a local store who can get her 6 recycling bins for $14.99. Elsie thinks her school needs 30 recycling bins. Set up and solve a proportion to find out how much money Elsie needs in order to buy all of the bins for her school. Remember to check your answer.

c. Now put it all together: Assuming Elsie recycles about 2070 cans each day, for how many days will Elsie have to recycle before she can buy her school new recycling bins?

5-75. Jeremy enlarged the shape at right to create a similar shape. The side that was originally 34 units long became 51 units long. How long is the side that was originally 10 units long?



5-76. Beth's favorite toy is a 4-inch-long scale model of a popular convertible. The full-sized convertible is 184 inches long and 74 inches wide. Use a proportion to find the width of Beth's model.

MATH NOTES

LOOKING DEEPER

Rational Numbers and Closure

Any number that can be written in the form $\frac{a}{b}$ (with a and b being integers and b not being zero) is called a **rational number**.

For example, –5, $2\frac{3}{4}$, and $0.\overline{6}$ are all rational numbers, as illustrated below.

$-5 = \frac{-5}{1}$ $\qquad$ $2\frac{3}{4} = \frac{11}{4}$ $\qquad$ $0.\overline{6} = \frac{2}{3}$

A set of numbers is called **closed** under an operation (like addition or multiplication) when using that operation with some of those numbers always results in one of those kinds of numbers. For example, odd numbers are closed under multiplication since $(odd)\cdot(odd) = odd$, but are not closed under addition since $(odd)+(odd) \neq odd$. The **closure properties** of rational numbers state that for all rational numbers a and b, $a+b$ and $a \cdot b$ are both rational numbers.

Review & Preview

5-77. Solve each equation below for the indicated variable, if possible. Show all steps.

a. Solve for x: $2x + 22 = 12$

b. Solve for y: $2x - y = 3$

c. Solve for x: $2x + 15 = 2x - 15$

d. Solve for y: $6x + 2y = 10$

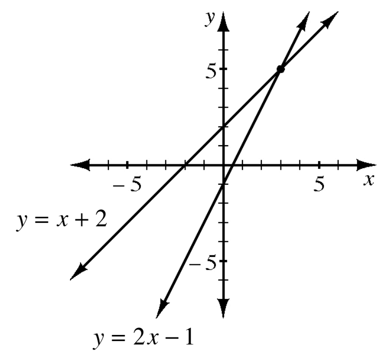
5-78. Janelle came to bat 464 times in 131 games. At this rate, how many times should she expect to have at bat in a full season of 162 games?

5-79. Jung's car travels 32 miles per gallon of gas. For each question below, write an equation, and then solve it.

a. How far will Jung's car go on 8 gallons of gas?

b. If Jung drives 118 miles, how much gas will be used?

5-80. The graph at right contains the lines for $y = x + 2$ and $y = 2x - 1$.



a. Using the graph, what is the solution to this system?

b. Solve the system algebraically to confirm your answer to part (a).

5-81. The Math Notes box for this lesson explains that the set (or group) of odd numbers is closed under multiplication because (odd)(odd) = odd. However, the set of odd numbers is *not* closed under addition because (odd) + (odd) ≠ odd.

a. Examine the set of all even numbers. Is this set closed under addition? Show how you know.

b. Is the set of even numbers closed under multiplication? Show how you know.

5-82. For each generic rectangle below, find the dimensions (length and width). Then write the area as a **product** of the dimensions and as a **sum**.

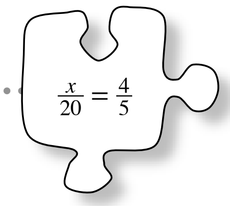
a.

$2x^2$	$10x$

b.

$2x^2$	$10x$
$3x$	15

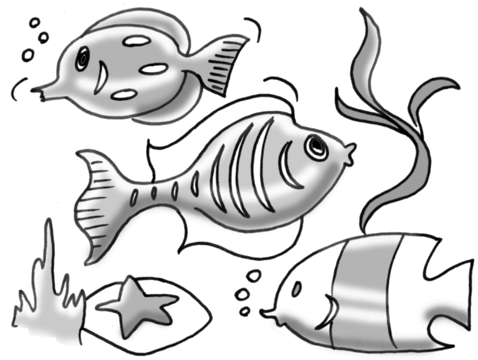
5.2.3 How can I use proportionality?



Applying Proportions

5-83. ESTIMATING FISH POPULATIONS TEAM CHALLENGE

Fish biologists need to keep track of fish populations in the waters they monitor. They want to know, for example, how many striped bass there are in San Francisco Bay. This number changes throughout the year, however, as fish move in and out of the bay to spawn. Therefore, biologists need a way to gather current data fairly quickly and inexpensively.



Your team will be given a "lake" (paper bag) with "fish" (beans). How many fish are in your lake?

Your Task: Determine the number of fish in your lake as accurately as possible *without* actually counting the fish. Then count the fish and find out how accurate your method was. Be ready to share your process and solution with the class.

Discussion Points

What are you supposed to find? Explain in your own words.

How do you think fish biologists determine the population of fish in a lake?

What information can you gather to help you answer this question?

What tools will you need?

Can you use a proportion to determine the number of fish in your lake?
Why or why not?

Further Guidance

5-84. Since it is impossible to count every animal, biologists use a process called "tag and recapture" to help them estimate the size of a population. Tag and recapture involves collecting a sample of animals, tagging them, and releasing them back into the wild. Later, biologists collect a new sample of the animals and count the number in the sample, distinguishing between first-time captures and recaptures. Then they use the data to estimate the population size.

Your team's task is to use the tag-and-recapture process to estimate the number of "fish" (beans) in your "lake" (paper bag).

a. How many fish do you think are in your lake? Estimate.

b. Use the "net" (small cup) to collect an initial sample. Carefully count the number of fish in the sample and record the data on your paper.

c. In order to tag the fish, replace each fish in the sample with a fish of a different color. Add these tagged fish to the lake. Be careful not to let any of the fish jump out onto the floor! (Put the original fish from your sample aside. Do not return them to the lake, or else this will increase the number of fish in the lake.)

d. Gently shake the bag to mix the fish thoroughly. Then collect another sample. Count the number of tagged and untagged fish in this new sample and record the information on your paper. Then return the entire sample to the lake.

e. Look over the data you have collected. How many tagged fish are in the lake? How many tagged fish were in the second sample? What was the total number of fish in the second sample? Use this data to determine the total number of fish in the lake.

f. Repeat the process outlined in parts (c) and (d) to get a second estimate of the total number of fish in the lake. Is this second estimate close to the first?

g. **Extension:** Your solutions represent two estimates for the fish population of your lake. While it is important to get an accurate count, each time you net a sample, it costs the taxpayers $500 for your time and equipment. So far, your samples have cost a total of $1000. If you think your estimate is accurate at this point, record it on the class chart with your cost. If you think you should try another sample for better accuracy, do the same steps as before. Draw as many samples as you need, but remember that each sample costs $500.

h. Count the fish in your lake to find the actual population. Then record your team's data on the class chart. Use the average of your estimates to represent your overall estimate of fish in the lake.

i. Was your estimate close? Was it better than your estimate from part (a)? If not, what might have thrown it off? Is this method of counting populations accurate? Why or why not?

Further Guidance section ends here.

MATH NOTES

METHODS AND MEANINGS

Using Generic Rectangles to Multiply

A generic rectangle can be used to find products because it helps to organize the different areas that make up the total rectangle. For example, to multiply $(2x+5)(x+3)$, a generic rectangle can be set up and completed as shown below. Notice that each product in the generic rectangle represents the area of that part of the rectangle.

	$2x$	$+5$
x		
$+3$		

$\longrightarrow$

	$2x$	$+5$
x	$2x^2$	$5x$
$+3$	$6x$	15

$$\underbrace{(2x+5)(x+3)}_{\text{area as a product}} = \underbrace{2x^2+11x+15}_{\text{area as a sum}}$$

Note that while a generic rectangle helps organize the problem, its size and scale is not important.

Review & Preview

5-85. Find each of the following products by drawing and labeling a generic rectangle or by using the Distributive Property.

a. $(x+2)(x+8)$

b. $(2m+30)(m+20)$

c. $x(y+10)$

d. $(2x+3)(3x+4)$

5-86. Did you know that the Statue of Liberty was a gift from France? It was shipped to New York and reassembled on an island in New York Harbor. It was finished in 1886. The distance from the base to the torch is 152 feet. The gift store sells a scale model of the statue measuring 18 inches (1.5 feet) tall.

a. If the length of the index finger on the real statue is eight feet, what is its length on the scale model?

b. Alex wanted to know the length of the right arm on the statue. He measured the model, and the right arm was five inches long. What is the length of the arm on the statue?

5-87. Solve each of the following equations for x. Then check each solution.

a. $\frac{x}{16} = \frac{7}{10}$ b. $\frac{6}{15} = \frac{3}{x}$ c. $\frac{2x}{5} = \frac{12}{8}$ d. $-8 = \frac{2}{x}$

5-88. Graph the lines $y = -4x + 3$ and $y = x - 7$ on the same set of axes. Then find their point of intersection.

5-89. Change each equation below into $y = mx + b$ form.

a. $y - 4x = -3$

b. $3y - 3x = 9$

c. $3x + 2y = 12$

d. $2(x - 3) + 3y = 0$

Chapter 5 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect on what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for **connections** between ideas as well as **connections** with material you learned previously.

① TEAM BRAINSTORM

With your team, brainstorm a list for each of the following topics. Be as detailed as you can. How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Topics: What have you studied in this chapter? What ideas and words were important in what you learned? Remember to be as detailed as you can.

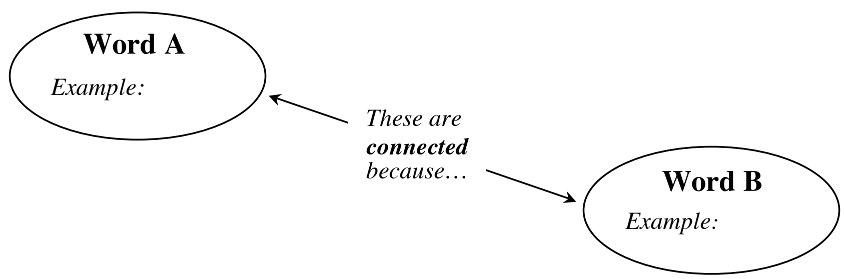
Connections: What topics, ideas, and words that you learned *before* this chapter are **connected** to the new ideas in this chapter? Again, make your list as long as you can.

② MAKING CONNECTIONS

The following is a list of the vocabulary used in this chapter. The words that appear in bold are new to this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

area	dimensions	**Distributive Property**
generic rectangles	growth	"legal" moves
linear equation	product	**proportion**
ratio	similar	solution
solve	**standard form**	starting value
sum	$y = mx + b$	

Make a concept map showing all of the **connections** you can find among the key words and ideas listed above. To show a **connection** between two words, draw a line between them and explain the **connection**, as shown in the example below. A word can be **connected** to any other word as long as there is a **justified connection**. For each key word or idea, provide a sketch that illustrates the idea (see the example below).



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the **connections** explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed above. Be sure to include these ideas on your concept map.

③ SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this.

④ WHAT HAVE I LEARNED?

This section will help you evaluate which types of problems you have seen with which you feel comfortable and those with which you need more help. This section appears at the end of every chapter to help you check your understanding. Even if your teacher does not assign this section, it is a good idea to try the problems and find out for yourself what you know and what you need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like these.

CL 5-90. Two brothers, Martin and Morris, are in their backyard. Morris is taking down a wall on one side of the yard while Martin is building a wall on the other side. Martin starts from scratch and lays 2 bricks every minute. Meanwhile, Morris takes down 3 bricks each minute from his wall. It takes Morris 55 minutes to finish tearing down his wall.

a. How many bricks were originally in the wall that Morris started tearing down?

b. Represent this situation with equations, tables, and a graph.

c. When did the two walls have the same number of bricks?

CL 5-91. Rewrite each of these products as a sum.

a. $6x(2x+y-5)$

b. $(2x-11)(x+4)$

c. $(7x)(2xy)$

d. $(x-2)(3+y)$

CL 5-92. Find the missing areas and dimensions for each generic rectangle below. Then write each area as a sum and as a product.

a.

$8x$	$16x^2$	
		1

b.

$10y$	
	$-12x$
$2y$	-3

CL 5-93. For each equation below, solve for x.

a. $(x-1)(x+7)=(x+1)(x-3)$

b. $2x-5(x+4)=-2(x+3)$

CL 5-94. For each equation below, solve for y.

a. $6x - 2y = 4$

b. $6x + 3y = 4x - 2y + 8$

c. Find the growth factors and y-intercepts for the equations in parts (a) and (b).

CL 5-95. For every 42 berries Samantha picks, her dog Clepto eats 7 berries. Samantha picked 462 berries last Saturday.

a. How many berries did Clepto eat last Saturday? Answer this question by writing and solving a proportion.

b. After Clepto was finished eating, how many berries did Samantha take home on Saturday?

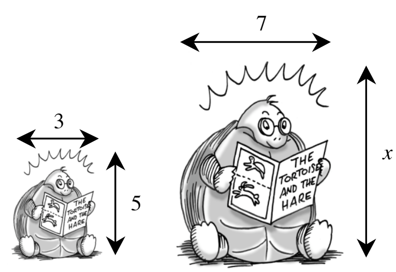
CL 5-96. Solve for each variable.

a. $\frac{x}{7} = \frac{3}{10}$

b. $\frac{8}{m} = \frac{3}{22}$

c. $\frac{11}{5} = \frac{2p}{3}$

CL 5-97. Kirstin enlarged her favorite picture at right on her computer so that the enlarged figure was similar to her original drawing. If the measurements of the original and new figure are as shown in the diagram at right, find x. Show all work.



CL 5-98. Find x and y for the system of equations at right:

$$y = 3x - 5$$
$$y = -x + 23$$

CL 5-99. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

⑤ HOW AM I THINKING?

This course focuses on five different **Ways of Thinking**: reversing thinking, justifying, generalizing, making connections, and applying and extending understanding. These are some of the ways in which you think while trying to make sense of a concept or to solve a problem (even outside of math class). During this chapter, you have probably used each Way of Thinking multiple times without even realizing it!

So far, each chapter of this course has focused on a different Way of Thinking, as you can see in the table below.

Chapter 1	**making connections**
Chapter 2	**justifying**
Chapter 3	**generalizing**
Chapter 4	**reversing thinking**

This closure activity will focus on the fifth Way of Thinking: **applying and extending**. Read the description of this Way of Thinking at right.

Think about the topics that you have learned during this chapter. When did you broaden your understanding of a concept? When did you apply an idea to solve a real-life problem? You may want to flip through the chapter to refresh your memory about the problems that you have worked on. Discuss any ideas you have with the rest of the class.

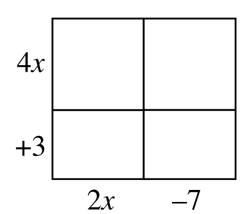
Once your discussion is complete, examine some of the ways you have **applied** and **extended** your understanding as you answer the questions below.

Applying and Extending

To extend understanding means to increase or expand what you know about an idea. You think this way when you try to apply your knowledge in new ways or consider new possibilities. An application is often the answer to a question like, "*How can I use this?*" When you catch yourself thinking, "*What if …*", you are usually trying to extend your understanding.

a. One way you **applied** your understanding of area was to use an area model to multiply expressions. For example, the area of the rectangle at right represents the product $(4x+3)(2x-7)$.

   i. Copy and complete the generic rectangle and write its area as a sum.



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⑤ *Continued from previous page.*

ii. Now **apply** your understanding to find two more products similar to $(4x+3)(2x-7)$. In other words, create your own products in the form $(ax+b)(cx+d)$ and use an area model to multiply.

iii. Now **extend** this idea: What if one of the expressions being multiplied has three terms? How can a generic rectangle be used to multiply two expressions such as $(x-3)(3y+2x+1)$? Discuss this with your team. Then create and complete a generic rectangle for $(x-3)(3y+2x+1)$ and write its area as a sum.

iv. Part (*iii*) was an **extension** because it considered a new "What if…?" question that came from the study of multiplying expressions. Now, as a team, write your own "What if…?" questions that come from this work. Be ready to share your questions with the class.

b. Now examine how you can **apply** proportional reasoning to solve different problems. With your team, answer the questions below.

i. The math club is sponsoring a math contest, and to prepare, Clarisse needs to sharpen 568 pencils. Luckily, the club has an electric pencil sharpener! When Clarisse started to sharpen the first 8 pencils, she noticed that it took her 2.5 minutes. Assuming she can continue sharpening pencils at the same rate, how long will it take her to sharpen the rest of the pencils? Write and solve a proportion to answer this question.

ii. To pay for trophies, the math club will sell 176 raffle tickets for $1 each. The club will randomly select 22 tickets to award prizes. If the club sponsor, Mr. Wallis, bought 40 tickets, will he probably win a prize? If so, how many prizes do you predict he would win? Show how you found our answer.

iii. Now, with your team, create at least two problems that would require someone to **apply** proportional reasoning to solve. Be creative! Be ready to share your problems with the class.

Answers and Support for Closure Activity #4
What Have I Learned?

Problem	Solution	Need Help?	More Practice
CL 5-90.	a. 55(3) = 165 bricks	Lesson 5.1.5 Math Notes box	Problems 5-19, 5-31, 5-42, 5-45, 5-61, and 5-68

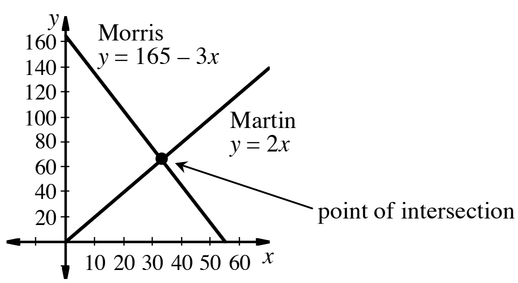
b. Martin's rule: $y = 2x$
Morris's rule: $y = 165 - 3x$

Martin's table:

Min.	Bricks
0	0
1	2
2	4
…	…
56	112
57	114

Morris's table:

Min.	Bricks
0	165
1	162
2	159
…	…
54	3
55	0



c. After 33 hours, they will each have 66 bricks.

Problem	Solution	Need Help?	More Practice
CL 5-91.	a. $12x^2 + 6xy - 30x$ b. $2x^2 - 3x - 44$ c. $14x^2y$ d. $3x + xy - 6 - 2y$	Lessons 5.1.1 and 5.1.3 Math Notes boxes	Problems 5-11, 5-13, 5-21, 5-25, 5-27, 5-33, 5-50, 5-70, and 5-85

Problem	Solution	Need Help?	More Practice
CL 5-92.	a. $(2x+1)(1+8x)=16x^2+10x+1$ $\begin{array}{c\|c\|c\|}\cline{2-3} 8x & 16x^2 & 8x \\ \cline{2-3} 1 & 2x & 1 \\ \cline{2-3} \multicolumn{1}{c}{} & \multicolumn{1}{c}{2x} & \multicolumn{1}{c}{1} \end{array}$ b. $(2y-3)(4x+5)=$ $8xy-12x+10y-15$ $\begin{array}{c\|c\|c\|}\cline{2-3} 5 & 10y & -15 \\ \cline{2-3} 4x & 8xy & -12x \\ \cline{2-3} \multicolumn{1}{c}{} & \multicolumn{1}{c}{2y} & \multicolumn{1}{c}{-3} \end{array}$	Lessons 5.1.1, 5.1.3, and 5.2.3 Math Notes boxes	Problems 5-16, 5-24, 5-26, 5-58, and 5-82
CL 5-93.	a. $x=\frac{1}{2}$ b. $x=-14$	Lesson 5.1.3 Math Notes box, Lesson 5.1.4	Problems 5-34, 5-37, 5-38, 5-48, 5-55, and 5-57
CL 5-94.	a. $y=3x-2$ b. $y=-\frac{2}{5}x+\frac{8}{5}$ c. part (a): $m=3$, $b=-2$ part (b): $m=-\frac{2}{5}$, $b=\frac{8}{5}$	Lesson 5.1.5	Problems 5-46, 5-47, 5-48, 5-51, 5-55, 5-77, and 5-89
CL 5-95.	a. 77 berries b. 385 berries	Lesson 5.2.1 Math Notes box	Problems 5-54, 5-62, 5-66, 5-67, 5-75, 5-76, 5-78, 5-79, and 5-86

Problem	Solution	Need Help?	More Practice
CL 5-96.	a. $x=\frac{21}{10}$ or 2.1 b. $x=\frac{176}{3}$ or $58\frac{2}{3}$ c. $p=\frac{33}{10}$ or 3.3	Section 5.2	Problems 5-17, 5-28, 5-63, 5-72, and 5-87
CL 5-97.	$x=\frac{35}{3}\approx 11.7$	Lesson 5.2.1 Math Notes box	Problems 5-54, 5-62, 5-66, 5-67, 5-75, 5-76, 5-78, 5-79, and 5-86
CL 5-98.	$x=7$ $y=16$	Lesson 4.2.1 Math Notes box, Lesson 4.2.3, problem 4-86, and Lesson 4.2.4 Math Notes box	Problems 5-43, 5-53, 5-80, and 5-88